Optimized localization and hybridization to filter ensemble-based covariances

Benjamin Ménétrier and Tom Auligné
NCAR - Boulder - Colorado

Roanoke - 06/04/2015
Introduction

Context:

• DA often relies on forecast error covariances.
• This matrix can be sampled from an ensemble of forecasts.
• Sampling noise arises because of the limited ensemble size.
• Question: how to filter this sampling noise?

Usual methods:

• Covariance localization → tapering with a localization matrix
• Covariance hybridization → linear combination with a static covariance matrix
Introduction

Context:

- DA often relies on forecast error covariances.
Introduction

Context:

- DA often relies on forecast error covariances.
- This matrix can be sampled from an ensemble of forecasts.
Introduction

Context:

• DA often relies on forecast error covariances.
• This matrix can be sampled from an ensemble of forecasts.
• Sampling noise arises because of the limited ensemble size.
Introduction

Context:

- DA often relies on forecast error covariances.
- This matrix can be sampled from an ensemble of forecasts.
- Sampling noise arises because of the limited ensemble size.
- Question: how to filter this sampling noise?
Introduction

Context:
- DA often relies on forecast error covariances.
- This matrix can be sampled from an ensemble of forecasts.
- Sampling noise arises because of the limited ensemble size.
- Question: how to filter this sampling noise?

Usual methods:
Introduction

Context:
- DA often relies on forecast error covariances.
- This matrix can be sampled from an ensemble of forecasts.
- Sampling noise arises because of the limited ensemble size.
- Question: how to filter this sampling noise?

Usual methods:
- Covariance localization → tapering with a localization matrix
## Introduction

**Context:**
- DA often relies on forecast error covariances.
- This matrix can be sampled from an ensemble of forecasts.
- Sampling noise arises because of the limited ensemble size.
- Question: how to filter this sampling noise?

**Usual methods:**
- Covariance localization
  - tapering with a localization matrix
- Covariance hybridization
  - linear combination with a static covariance matrix
# Introduction

## Questions:

1. Can localization and hybridization be considered together?
2. Is it possible to optimize localization and hybridization coefficients objectively and simultaneously?
3. Is hybridization always improving the accuracy of forecast error covariances?
Introduction

Questions:

1. Can localization and hybridization be considered together?
Questions:

1. Can localization and hybridization be considered together?

2. Is it possible to optimize localization and hybridization coefficients objectively and simultaneously?
Introduction

Questions:

1. Can localization and hybridization be considered together?

2. Is it possible to optimize localization and hybridization coefficients objectively and simultaneously?

The method should:
Introduction

Questions:

1. Can localization and hybridization be considered together?

2. Is it possible to optimize localization and hybridization coefficients objectively and simultaneously?

The method should:
Questions:

1. Can localization and hybridization be considered together?

2. Is it possible to optimize localization and hybridization coefficients objectively and simultaneously?

The method should:

- use data from the ensemble only.
Questions:

1. Can localization and hybridization be considered **together**?

2. Is it possible to optimize localization and hybridization coefficients **objectively and simultaneously**?

The method should:

- use data from the **ensemble only**.
- be affordable for **high-dimensional systems**.
Introduction

Questions:

1. Can localization and hybridization be considered together?

2. Is it possible to optimize localization and hybridization coefficients objectively and simultaneously?

The method should:

- use data from the ensemble only.
- be affordable for high-dimensional systems.

3. Is hybridization always improving the accuracy of forecast error covariances?
Outline

Introduction

Linear filtering of sample covariances

Joint optimization of localization and hybridization

Results

Conclusions
## Outline

<table>
<thead>
<tr>
<th>Introduction</th>
<th>Linear filtering</th>
<th>Joint optimization</th>
<th>Results</th>
<th>Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introduction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear filtering of sample covariances</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint optimization of localization and hybridization</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Results</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Conclusions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Linear filtering of sample covariances

An ensemble of \( N \) forecasts \( \{ \tilde{x}_b^p \} \) is used to sample \( \tilde{B} \):

\[
\tilde{B} = \frac{1}{N-1} \sum_{p=1}^{N} (\tilde{x}_b^p - \langle \tilde{x}_b \rangle)(\tilde{x}_b^p - \langle \tilde{x}_b \rangle)^T
\]

where:

\[
\delta \tilde{x}_b^p = \tilde{x}_b^p - \langle \tilde{x}_b \rangle
\]

\[
\langle \tilde{x}_b \rangle = \frac{1}{N} \sum_{p=1}^{N} \tilde{x}_b^p
\]

Asymptotic behavior: if \( N \to \infty \), then \( \tilde{B} \to \tilde{B}^\star \)

In practice, \( N < \infty \) ⇒ sampling noise

Theory of sampling error:

\[
E[\tilde{B}_{ij}^2] = \frac{N(N-3)}{(N-1)^2} E[\tilde{B}^{\star 2}_{ij}] - \frac{1}{(N-1)(N-2)} E[\tilde{B}_{ii}\tilde{B}_{jj}] + \frac{N^2}{(N-1)^2(N-2)} E[\tilde{\Xi}_{ijij}]
\]
Linear filtering of sample covariances

An ensemble of $N$ forecasts $\{\tilde{x}^b_p\}$ is used to sample $\tilde{B}$:

$$\tilde{B} = \frac{1}{N-1} \sum_{p=1}^{N} \delta\tilde{x}^b (\delta\tilde{x}^b)^T$$

where:  
$$\delta\tilde{x}^b = \tilde{x}^b - \langle \tilde{x}^b \rangle \quad \text{and} \quad \langle \tilde{x}^b \rangle = \frac{1}{N} \sum_{p=1}^{N} \tilde{x}^b_p$$
Linear filtering of sample covariances

An ensemble of $N$ forecasts $\{\tilde{x}_p^b\}$ is used to sample $\tilde{B}$:

$$\tilde{B} = \frac{1}{N-1} \sum_{p=1}^{N} \delta\tilde{x}^b (\delta\tilde{x}^b)^T$$

where:

$$\delta\tilde{x}_p^b = \tilde{x}_p^b - \langle \tilde{x}^b \rangle \quad \text{and} \quad \langle \tilde{x}^b \rangle = \frac{1}{N} \sum_{p=1}^{N} \tilde{x}_p^b$$

Asymptotic behavior: if $N \to \infty$, then $\tilde{B} \to \tilde{B}^*$
Linear filtering of sample covariances

An ensemble of $N$ forecasts $\{\tilde{x}_p^b\}$ is used to sample $\tilde{B}$:

$$
\tilde{B} = \frac{1}{N-1} \sum_{p=1}^N \delta\tilde{x}^b (\delta\tilde{x}^b)^T
$$

where:

$$
\delta\tilde{x}_p^b = \tilde{x}_p^b - \langle \tilde{x}^b \rangle \quad \text{and} \quad \langle \tilde{x}^b \rangle = \frac{1}{N} \sum_{p=1}^N \tilde{x}_p^b
$$

Asymptotic behavior: if $N \to \infty$, then $\tilde{B} \to \tilde{B}^*$

In practice, $N < \infty \Rightarrow$ sampling noise $\tilde{B}^e = \tilde{B} - \tilde{B}^*$
Linear filtering of sample covariances

An ensemble of $N$ forecasts $\{\tilde{x}^b_p\}$ is used to sample $\tilde{B}$:

$$\tilde{B} = \frac{1}{N-1} \sum_{p=1}^{N} \delta\tilde{x}^b(\delta\tilde{x}^b)^T$$

where:

$$\delta\tilde{x}^b_p = \tilde{x}^b_p - \langle \tilde{x}^b \rangle$$

and

$$\langle \tilde{x}^b \rangle = \frac{1}{N} \sum_{p=1}^{N} \tilde{x}^b_p$$

Asymptotic behavior: if $N \to \infty$, then $\tilde{B} \to \tilde{B}^*$

In practice, $N < \infty \Rightarrow$ sampling noise $\tilde{B}^e = \tilde{B} - \tilde{B}^*$

Theory of sampling error:

$$\mathbb{E}[\tilde{B}_{ij}^2] = \frac{N(N-3)}{(N-1)^2} \mathbb{E}[\tilde{B}_{ij}^*^2] - \frac{1}{(N-1)(N-2)} \mathbb{E}[\tilde{B}_{ii} \tilde{B}_{jj}]$$

$$+ \frac{N^2}{(N-1)^2(N-2)} \mathbb{E}[\tilde{\Xi}_{jjj}]$$
Linear filtering of sample covariances

Localization by $L$ (Schur product)

Covariance matrix

$$\hat{B} = L \circ \tilde{B}$$
Linear filtering of sample covariances

\[
\hat{\mathbf{B}} = \mathbf{L} \circ \tilde{\mathbf{B}}
\]

\[
\delta \mathbf{x}^e = \frac{1}{\sqrt{N-1}} \sum_{p=1}^{N} \delta \tilde{\mathbf{x}}^b_p \circ (\mathbf{L}^{1/2} \mathbf{v}^\alpha_p)
\]
Linear filtering of sample covariances

Localization by $\mathbf{L}$ (Schur product)

Covariance matrix

$$\hat{\mathbf{B}} = \mathbf{L} \circ \tilde{\mathbf{B}}$$

Increment

$$\delta \mathbf{x}^e = \frac{1}{\sqrt{N-1}} \sum_{p=1}^{N} \delta \tilde{\mathbf{x}}^b_p \circ (\mathbf{L}^{1/2} \mathbf{v}^\alpha_p)$$

Localization by $\mathbf{L}$ + hybridization with $\mathbf{\bar{B}}$

Increment

$$\delta \mathbf{x} = \beta^e \delta \mathbf{x}^e + \beta^c \mathbf{\bar{B}}^{1/2} \mathbf{v}^c$$
Linear filtering of sample covariances

**Localization by \( L \) (Schur product)**

Covariance matrix

\[
\hat{B} = L \circ \tilde{B}
\]

Increment

\[
\delta x^e = \frac{1}{\sqrt{N - 1}} \sum_{p=1}^{N} \delta \tilde{x}_b \circ (L^{1/2} v^\alpha_p)
\]

**Localization by \( L + \) hybridization with \( \bar{B} \)**

Covariance matrix

\[
\hat{B}^h = (\beta^e)^2 L \circ \tilde{B} + (\beta^c)^2 \bar{B}
\]

Increment

\[
\delta x = \beta^e \delta x^e + \beta^c \bar{B}^{1/2} v^c
\]
Linear filtering of sample covariances

Localization by $L$ (Schur product)

Covariance matrix

$$\hat{\mathbf{B}} = L \circ \tilde{\mathbf{B}}$$

Increment

$$\delta x^e = \frac{1}{\sqrt{N - 1}} \sum_{p=1}^{N} \delta \tilde{x}_p^b \circ (L^{1/2} \mathbf{v}_p^\alpha)$$

Localization by $L +$ hybridization with $\mathbf{B}$

Covariance matrix

$$\hat{\mathbf{B}}^h = (\beta^e)^2 L \circ \tilde{\mathbf{B}} + (\beta^c)^2 \mathbf{B}$$

Gain $L^h$

Increment

$$\delta x = \beta^e \delta x^e + \beta^c \mathbf{B}^{1/2} \mathbf{v}^c$$

Offset
Linear filtering of sample covariances

Localization by \( L \) (Schur product)

Covariance matrix
\[
\hat{B} = L \circ \tilde{B}
\]

Increment
\[
\delta x^e = \frac{1}{\sqrt{N-1}} \sum_{p=1}^{N} \delta \tilde{x}_p^b \circ \left( L^{1/2} \nu^\alpha_p \right)
\]

Localization by \( L \) + hybridization with \( \bar{B} \)

Covariance matrix
\[
\hat{B}^h = (\beta^e)^2 L \circ \tilde{B} + (\beta^c)^2 \bar{B}
\]

Gain \( L^h \)

Offset

Increment
\[
\delta x = \beta^e \delta x^e + \beta^c \bar{B}^{1/2} \nu^c
\]

Localization + hybridization = linear filtering of \( \tilde{B} \)

\( L^h \) and \( \beta^c \) have to be optimized together
Outline

Introduction

Linear filtering of sample covariances

Joint optimization of localization and hybridization

Results

Conclusions
Joint optimization: step 1

Step 1: optimizing the **localization only**, without hybridization
Joint optimization: step 1

Step 1: optimizing the **localization only**, without hybridization

Goal: to minimize the expected quadratic error:

\[
e = \mathbb{E} \left[ \| L \odot \tilde{B} - \tilde{B}^* \|^2 \right]
\]

\[\text{Localized } \tilde{B} \quad \text{Asymptotic } \tilde{B}\] (1)

Light assumptions:
- The unbiased sampling noise \( \tilde{B} - \tilde{B}^* \) is not correlated with the asymptotic sample covariance matrix \( \tilde{B}^* \).
- The two random processes generating the asymptotic \( \tilde{B}^* \) and the sample distribution are independent.
Joint optimization: step 1

Step 1: optimizing the **localization only**, without hybridization

Goal: to minimize the expected quadratic error:

\[
e = E \left[ \| L \circ \tilde{B} - \tilde{B}^* \|^2 \right]
\]

Light assumptions:

- The unbiased sampling noise \( \tilde{B} = \tilde{B} - \tilde{B}^* \) is not correlated with the asymptotic sample covariance matrix \( \tilde{B}^* \).
- The two random processes generating the asymptotic \( \tilde{B}^* \) and the sample distribution are independent.

An explicit formula for the optimal localization \( L \) is given in Ménétrier et al. 2015 (Montly Weather Review).
Joint optimization: step 1

Step 1: optimizing the localization only, without hybridization

Goal: to minimize the expected quadratic error:

\[ e = \mathbb{E} \left[ \| L \circ \tilde{B} - \tilde{B}^* \|^2 \right] \]  \hspace{1cm} (1)

Localized \( \tilde{B} \)  \hspace{1cm} Asymptotic \( \tilde{B} \)

Light assumptions:

- The unbiased sampling noise \( \tilde{B}^e = \tilde{B} - \tilde{B}^* \) is not correlated with the asymptotic sample covariance matrix \( \tilde{B}^* \).
Joint optimization: step 1

Step 1: optimizing the **localization only**, without hybridization

Goal: to minimize the expected quadratic error:

\[
e = \mathbb{E} \left[ \| L \circ \tilde{B} - \tilde{B}^* \|^2 \right]
\]

Light assumptions:

- The unbiased sampling noise \( \tilde{B}^e = \tilde{B} - \tilde{B}^* \) is not correlated with the asymptotic sample covariance matrix \( \tilde{B}^* \).
- The two random processes generating the asymptotic \( \tilde{B}^* \) and the sample distribution are independent.
Joint optimization: step 1

Step 1: optimizing the **localization only**, without hybridization

Goal: to minimize the expected quadratic error:

\[
e = \mathbb{E} \left[ \| L \circ \tilde{B} - \tilde{B}^* \|^2 \right]
\]

(1)

Light assumptions:

- The unbiased sampling noise \( \tilde{B}^e = \tilde{B} - \tilde{B}^* \) is not correlated with the asymptotic sample covariance matrix \( \tilde{B}^* \).
- The two random processes generating the asymptotic \( \tilde{B}^* \) and the sample distribution are independent.

An **explicit formula** for the optimal localization \( L \) is given in Ménétrier et al. 2015 (Montly Weather Review).
Joint optimization: step 1

This formula of optimal localization $L$ involves:

- the ensemble size $N$
- the sample covariance $\tilde{B}$
- the sample fourth-order centered moment $\tilde{\Xi}$
Joint optimization: step 1

This formula of optimal localization $L$ involves:

- the ensemble size $N$
- the sample covariance $\tilde{B}$
- the sample fourth-order centered moment $\tilde{\Xi}$

\[
L_{ij} = \frac{(N - 1)^2}{N(N - 3)} - \frac{N}{(N - 2)(N - 3)} \frac{\mathbb{E}[\tilde{\Xi}_{ijij}]}{\mathbb{E}[\tilde{B}_{ij}^2]} + \frac{N - 1}{N(N - 2)(N - 3)} \frac{\mathbb{E}[\tilde{B}_{ij} \tilde{B}_{jj}]}{\mathbb{E}[\tilde{B}_{ij}^2]}
\]
Joint optimization: step 2

Step 2: optimizing localization and hybridization together
Joint optimization: step 2

Step 2: optimizing localization and hybridization together

Goal: to minimize the expected quadratic error

\[
e^h = \mathbb{E}[\|L^h \circ \tilde{B} + (\beta_c)^2 \overline{B} - \overline{B}^* \|^2]
\]

Localized / hybridized \( \tilde{B} \)  
Asymptotic \( \overline{B} \)
Joint optimization: step 2

Step 2: optimizing localization and hybridization together

Goal: to minimize the expected quadratic error

\[ e^h = \mathbb{E}[\| L^h \circ \tilde{B} + (\beta^c)^2 \tilde{B} - \tilde{B}^\star \|^2] \]

Localized / hybridized \( \tilde{B} \)  
Asymptotic \( \tilde{B} \)

Same assumptions as before.
Joint optimization: step 2

Step 2: optimizing localization and hybridization together

Goal: to minimize the expected quadratic error

\[ e^h = \mathbb{E} \left[ \left\| L^h \circ \tilde{B} + (\beta^c)^2 \tilde{B} \right\|^2 \right] - \tilde{B}^* \]

Localized / hybridized \( \tilde{B} \)  
Asymptotic \( \tilde{B}^* \)

Same assumptions as before.

Result of the minimization: a linear system in \( L^h \) and \( (\beta^c)^2 \)

\[
L^h_{ij} = L_{ij} - \frac{\mathbb{E}[\tilde{B}_{ij}]}{\mathbb{E}[\tilde{B}^2_{ij}]} \tilde{B}_{ij} (\beta^c)^2 \quad (2a)
\]

\[
(\beta^c)^2 = \frac{\sum_{ij} \tilde{B}_{ij} (1 - L^h_{ij}) \mathbb{E}[\tilde{B}_{ij}]}{\sum_{ij} \tilde{B}^2_{ij}} \quad (2b)
\]
Hybridization benefits

Comparison of:

• \( \hat{B} = L \circ \tilde{B} \), with an optimal \( L \) minimizing \( e \)

• \( \hat{B}^h = L^h \circ \tilde{B} + (\beta^c)^2 \bar{B} \), with optimal \( L^h \) and \( \beta^c \) minimizing \( e^h \)
Hybridization benefits

Comparison of:

- $\hat{\mathbf{B}} = \mathbf{L} \circ \tilde{\mathbf{B}}$, with an optimal $\mathbf{L}$ minimizing $e$
- $\hat{\mathbf{B}}^h = \mathbf{L}^h \circ \tilde{\mathbf{B}} + (\beta^c)^2 \tilde{\mathbf{B}}$, with optimal $\mathbf{L}^h$ and $\beta^c$ minimizing $e^h$

We can show that:

$$e^h - e = - (\beta^c)^2 \sum_{ij} \frac{\mathbb{B}_{ij}^2 \text{Var}(\tilde{\mathbb{B}}_{ij})}{\mathbb{E}[\tilde{\mathbb{B}}_{ij}^2]}$$

$\leq 0$ (3)
Hybridization benefits

Comparison of:

- \( \hat{B} = L \circ \tilde{B} \), with an optimal \( L \) minimizing \( e \)
- \( \hat{B}^h = L^h \circ \tilde{B} + (\beta^c)^2 \tilde{B} \), with optimal \( L^h \) and \( \beta^c \) minimizing \( e^h \)

We can show that:

\[
n^h - e = - (\beta^c)^2 \sum_{ij} \frac{\bar{B}_{ij}^2 \text{Var}(\tilde{B}_{ij})}{\mathbb{E}[\tilde{B}_{ij}^2]} \leq 0
\]

With optimal parameters, whatever the static \( \tilde{B} \):
Localization + hybridization is better than localization alone
Outline

Introduction

Linear filtering of sample covariances

Joint optimization of localization and hybridization

Results

Conclusions
Practical implementation

An ergodicity assumption is required to estimate the statistical expectations $\mathbb{E}$ in practice:
Practical implementation

An ergodicity assumption is required to estimate the statistical expectations \( \mathbb{E} \) in practice:

- whole domain average,
- local average,
- scale dependent average,
- etc.
Practical implementation

An ergodicity assumption is required to estimate the statistical expectations \( \mathbb{E} \) in practice:

- whole domain average,
- local average,
- scale dependent average,
- etc.

→ This assumption is independent from earlier theory.
Practical implementation

An ergodicity assumption is required to estimate the statistical expectations $\mathbb{E}$ in practice:

- whole domain average,
- local average,
- scale dependent average,
- etc.

→ This assumption is independent from earlier theory.

Localization $L^h$ and hybridization coefficient $\beta^c$ can be computed:

- from the ensemble at each assimilation window,
- climatologically from an archive of ensembles.
Practical implementation

An ergodicity assumption is required to estimate the statistical expectations $\mathbb{E}$ in practice:

- whole domain average,
- local average,
- scale dependent average,
- etc.

→ This assumption is independent from earlier theory.

Localization $L^h$ and hybridization coefficient $\beta^c$ can be computed:

- from the ensemble at each assimilation window,
- climatologically from an archive of ensembles.
Experimental setup

- WRF-ARW model, large domain, 25 km-resolution, 40 levels
Experimental setup

- WRF-ARW model, large domain, 25 km-resolution, 40 levels
- Initial conditions randomized from a homogeneous static B
Experimental setup

- **WRF-ARW model**, large domain, 25 km-resolution, 40 levels
- Initial conditions randomized from a homogeneous static B
- Reference and test ensembles (1000 / 100 members)
Experimental setup

- WRF-ARW model, large domain, 25 km-resolution, 40 levels
- Initial conditions randomized from a homogeneous static B
- Reference and test ensembles (1000 / 100 members)
- Forecast ranges: 12, 24, 36 and 48 h
Experimental setup

- **WRF-ARW model**, large domain, 25 km-resolution, 40 levels
- Initial conditions randomized from a homogeneous static B
- Reference and test ensembles (1000 / 100 members)
- Forecast ranges: 12, 24, 36 and 48 h

Temperature at level 7 (∼1 km above ground), 48 h-range forecasts
Localization and hybridization

- Optimization of the horizontal localization $L_{\text{hor}}^h$ and of the hybridization coefficient $\beta^c$ at each vertical level.
Localization and hybridization

- Optimization of the horizontal localization $L_{hor}^h$ and of the hybridization coefficient $\beta^c$ at each vertical level.
- Static $\overline{B} = \text{horizontal average of } \tilde{B}$
Localization and hybridization

- Optimization of the horizontal localization $L^h_{\text{hor}}$ and of the hybridization coefficient $\beta^c$ at each vertical level.
- Static $\bar{B} = \text{horizontal average of } \tilde{B}$
- Localization length-scale:

![Graphs showing localization and hybridization results](image-url)
Localization and hybridization

- Optimization of the horizontal localization $L_{\text{hor}}^h$ and of the hybridization coefficient $\beta^c$ at each vertical level.
- Static $\overline{B}$ = horizontal average of $\tilde{B}$
- Hybridization coefficients for zonal wind:

![Graph showing $eta^e$, $\beta^c$, and $(\beta^e)^2 + (\beta^c)^2$ at different u values.](image)
Localization and hybridization

- Optimization of the horizontal localization $L^h_{\text{hor}}$ and of the hybridization coefficient $\beta^c$ at each vertical level.
- Static $\overline{B} = \text{horizontal average of } \tilde{B}$
- Impact of the hybridization:
Localization and hybridization

- Optimization of the horizontal localization $\mathbf{L}^{h}_{\text{hor}}$ and of the hybridization coefficient $\beta^c$ at each vertical level.
- Static $\overline{\mathbf{B}} = \text{horizontal average of } \tilde{\mathbf{B}}$
- Impact of the hybridization:
  - $\tilde{\mathbf{B}}^*$ is estimated with the reference ensemble
Localization and hybridization

- Optimization of the horizontal localization $L^h_{\text{hor}}$ and of the hybridization coefficient $\beta^c$ at each vertical level.
- Static $\overline{B}$ = horizontal average of $\tilde{B}$
- Impact of the hybridization:
  - $\tilde{B}^*$ is estimated with the reference ensemble
  - Expected quadratic errors $e$ and $e^h$ are computed
Localization and hybridization

- Optimization of the horizontal localization $L^h_{\text{hor}}$ and of the hybridization coefficient $\beta^c$ at each vertical level.
- Static $\overline{B}$ = horizontal average of $\tilde{B}$
- Impact of the hybridization:
  - $\tilde{B}^*$ is estimated with the reference ensemble
  - Expected quadratic errors $e$ and $e^h$ are computed

Error reduction from $e$ to $e^h$ for 25 members

<table>
<thead>
<tr>
<th>Zonal wind</th>
<th>Meridian wind</th>
<th>Temperature</th>
<th>Specific humidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5 %</td>
<td>4.2 %</td>
<td>3.9 %</td>
<td>1.7 %</td>
</tr>
</tbody>
</table>
Localization and hybridization

- Optimization of the horizontal localization $L^h_{\text{hor}}$ and of the hybridization coefficient $\beta^c$ at each vertical level.
- Static $\overline{B}$ = horizontal average of $\tilde{B}$
- Impact of the hybridization:
  - $\tilde{B}^*$ is estimated with the reference ensemble
  - Expected quadratic errors $e$ and $e^h$ are computed

Error reduction from $e$ to $e^h$ for 25 members

<table>
<thead>
<tr>
<th>Zonal wind</th>
<th>Meridian wind</th>
<th>Temperature</th>
<th>Specific humidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5 %</td>
<td>4.2 %</td>
<td>3.9 %</td>
<td>1.7 %</td>
</tr>
</tbody>
</table>

→ Hybridization with $\overline{B}$ improves the accuracy of the forecast error covariance matrix
# Outline

<table>
<thead>
<tr>
<th>Introduction</th>
<th>Linear filtering</th>
<th>Joint optimization</th>
<th>Results</th>
<th>Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>Linear filtering of sample covariances</td>
<td>Joint optimization of localization and hybridization</td>
<td>Results</td>
<td>Conclusions</td>
</tr>
</tbody>
</table>
Conclusions

1. Localization and hybridization are two joint aspects of the linear filtering of sample covariances.

2. We have developed a new objective method to optimize localization and hybridization coefficients together:
   - Based on properties of the ensemble only
   - Affordable for high-dimensional systems
   - Tackling the sampling noise issue only

3. If done optimally, hybridization always improves the accuracy of forecast error covariances.

Ménétrier, B. and T. Auligné: Optimized Localization and Hybridization to Filter Ensemble-Based Covariances
Monthly Weather Review, 2015, accepted
Conclusions

1. Localization and hybridization are **two joint aspects** of the linear filtering of sample covariances.
Conclusions

1. Localization and hybridization are **two joint aspects** of the linear filtering of sample covariances.

2. We have developed a **new objective method** to optimize localization and hybridization coefficients together:
Conclusions

1. Localization and hybridization are **two joint aspects** of the linear filtering of sample covariances.

2. We have developed a **new objective method** to optimize localization and hybridization coefficients together:
   - Based on properties of the **ensemble only**
Conclusions

1. Localization and hybridization are **two joint aspects** of the linear filtering of sample covariances.

2. We have developed a **new objective method** to optimize localization and hybridization coefficients together:
   - Based on properties of the *ensemble only*
   - Affordable for *high-dimensional systems*
Conclusions

1. Localization and hybridization are **two joint aspects** of the linear filtering of sample covariances.

2. We have developed a **new objective method** to optimize localization and hybridization coefficients together:
   - Based on properties of the **ensemble only**
   - Affordable for **high-dimensional systems**
   - Tackling the sampling noise issue **only**
Conclusions

1. Localization and hybridization are two joint aspects of the linear filtering of sample covariances.

2. We have developed a new objective method to optimize localization and hybridization coefficients together:
   - Based on properties of the ensemble only
   - Affordable for high-dimensional systems
   - Tackling the sampling noise issue only

3. If done optimally, hybridization always improves the accuracy of forecast error covariances.
Conclusions

1. Localization and hybridization are **two joint aspects** of the linear filtering of sample covariances.

2. We have developed a **new objective method** to optimize localization and hybridization coefficients together:
   - Based on properties of the *ensemble only*
   - Affordable for **high-dimensional systems**
   - Tackling the sampling noise issue **only**

3. If done optimally, hybridization **always improves** the accuracy of forecast error covariances.

Ménétrier, B. and T. Auligné: Optimized Localization and Hybridization to Filter Ensemble-Based Covariances

*Monthly Weather Review, 2015*, accepted
Perspectives

Already done in the paper:

\[ \delta x \beta e \phi \delta x_{e} + \beta c \phi \delta x_{c} \rightarrow \]

Requires the solution of a nonlinear system

\[ A(L_{h}, \beta c) = 0, \]

performed by a bound-constrained minimization.

• Heterogeneous optimization: local averages over sub domains
• 3D optimization: joint computation of horizontal and vertical localizations, and hybridization coefficients

To be done:
• Tests in a cycled quasi-operational configuration
• Extension of the theory to account for systematic errors in \( \tilde{B} \star \) (theory is ready, tests are under way...)
Perspectives

Already done in the paper:

- Extension to vectorial hybridization weights:
  \[ \delta x = \beta^e \circ \delta x^e + \beta^c \circ \delta x^c \]
Perspectives

Already done in the paper:

• Extension to vectorial hybridization weights:

\[
\delta x = \beta^e \circ \delta x^e + \beta^c \circ \delta x^c
\]

→ Requires the solution of a nonlinear system \(A(L^h, \beta^c) = 0\), performed by a bound-constrained minimization.
Perspectives

Already done in the paper:

- Extension to vectorial hybridization weights:
  \[ \delta x = \beta^e \circ \delta x^e + \beta^c \circ \delta x^c \]

  \[ \rightarrow \text{Requires the solution of a nonlinear system } \mathcal{A}(L^h, \beta^c) = 0, \]
  performed by a bound-constrained minimization.

- Heterogeneous optimization: local averages over subdomains
Perspectives

Already done in the paper:

- Extension to vectorial hybridization weights:
  \[ \delta x = \beta^e \circ \delta x^e + \beta^c \circ \delta x^c \]

  \[ \rightarrow \text{Requires the solution of a nonlinear system } \mathcal{A}(L^h, \beta^c) = 0, \]
  performed by a bound-constrained minimization.

- Heterogeneous optimization: local averages over subdomains

- 3D optimization: joint computation of horizontal and vertical localizations, and hybridization coefficients
Perspectives

Already done in the paper:

- Extension to vectorial hybridization weights:
  \[ \delta x = \beta^e \circ \delta x^e + \beta^c \circ \delta x^c \]
  \[ \rightarrow \text{Requires the solution of a nonlinear system } \mathcal{A}(L^h, \beta^c) = 0, \]
  performed by a bound-constrained minimization.

- Heterogeneous optimization: local averages over subdomains

- 3D optimization: joint computation of horizontal and vertical localizations, and hybridization coefficients

To be done:
Perspectives

Already done in the paper:

- Extension to vectorial hybridization weights:
  \[ \delta x = \beta^e \circ \delta x^e + \beta^c \circ \delta x^c \]
  → Requires the solution of a nonlinear system \( A(L^h, \beta^c) = 0 \), performed by a bound-constrained minimization.

- Heterogeneous optimization: local averages over subdomains

- 3D optimization: joint computation of horizontal and vertical localizations, and hybridization coefficients

To be done:

- Tests in a cycled quasi-operational configuration
Perspectives

Already done in the paper:

- Extension to vectorial hybridization weights:
  \[ \delta x = \beta^e \circ \delta x^e + \beta^c \circ \delta x^c \]

  \[ \rightarrow \text{Requires the solution of a nonlinear system } A(L^h, \beta^c) = 0, \]
  \[ \text{performed by a bound-constrained minimization.} \]

- Heterogeneous optimization: local averages over subdomains

- 3D optimization: joint computation of horizontal and vertical localizations, and hybridization coefficients

To be done:

- Tests in a cycled quasi-operational configuration

- Extension of the theory to account for systematic errors in \( \tilde{B}^* \)
  \[ \text{(theory is ready, tests are underway...)} \]
Thank you for your attention!

Any question?